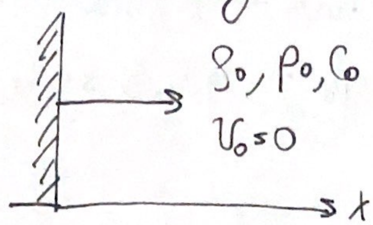
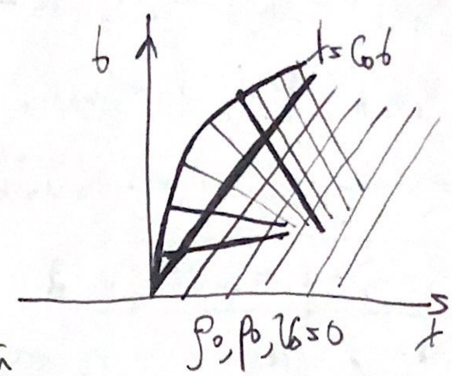
Задача о поршне вдвигаемом в газ

1) $V_{поршня} = \begin{cases} 0, & t=0 \\ V(t), & 0<t<t_0 \\ V_0, & t>t_0 \end{cases}$





Разрывные реш-я нестац-ых ур-ий газовой динамики.

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0 \ (1) \\ \rho\dfrac{\partial v}{\partial t} + \rho v\dfrac{\partial v}{\partial x} + \dfrac{\partial p}{\partial x} = 0 \ (2) \\ \rho\dfrac{\partial}{\partial t}\left(\varepsilon + \dfrac{v^2}{2}\right) + \rho v\dfrac{\partial}{\partial x}\left(\varepsilon + \dfrac{v^2}{2}\right) + \dfrac{\partial}{\partial x}(pv) = 0 \ (3) \end{cases}$$

$(1)\cdot v \Rightarrow v\dfrac{\partial \rho}{\partial t} + v\dfrac{\partial}{\partial x}(\rho v) = 0$ (2)

$\dfrac{\partial}{\partial t}(\rho v) + \dfrac{\partial}{\partial x}(\rho v^2) + \dfrac{\partial}{\partial x}p = 0$; $(1)\cdot\left(\varepsilon + \dfrac{v^2}{2}\right) \Rightarrow$

$\Rightarrow \left(\varepsilon + \dfrac{v^2}{2}\right)\dfrac{\partial \rho}{\partial t} + \left(\varepsilon + \dfrac{v^2}{2}\right)\dfrac{\partial}{\partial x}(\rho v) = 0$; $+(3)$

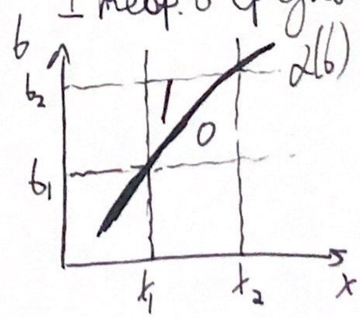
$\dfrac{\partial}{\partial t}\left(\rho\left(\varepsilon + \dfrac{v^2}{2}\right)\right) + \dfrac{\partial}{\partial x}\left(\rho v\left(\varepsilon + \dfrac{v^2}{2}\right) + pv\right) = 0$

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + \dfrac{\partial}{\partial x}(\rho v) = 0 \\ \dfrac{\partial}{\partial t}(\rho v) + \dfrac{\partial}{\partial x}(\rho v^2 + p) = 0 \\ \dfrac{\partial}{\partial t}\left(\rho\left(\varepsilon + \dfrac{v^2}{2}\right)\right) + \dfrac{\partial}{\partial x}\left(\rho v\left(\varepsilon + \dfrac{v^2}{2}\right) + pv\right) = 0 \end{cases}$$

$$\int_{x_1}^{x_2}(\rho(t_2) - \rho(t_1))\,dx + \int_{t_1}^{t_2}\left(\rho v\big|_{x_2} - \rho v\big|_{x_1}\right)dt = 0$$

$\Downarrow$

I теор. о среднем



$$\left(\rho(t_2)\big|_{x^*} - \rho(t_1)\big|_{x^{**}}\right)\Delta x + \left(\rho v\big|_{t^*}^{x_2} - \rho v\big|_{t^{**}}^{x_2}\right)\Delta t = 0$$

$x^* \in [x_1, x_2]$; $x^{**} \in [x_1, x_2]$; $t^* \in [t_1, t_2]$; $t^{**} \in [t_1, t_2]$

$(\rho_1 - \rho_0)\dfrac{\Delta x}{\Delta t} + \rho_0 v_0 - \rho_1 v_1 = 0$; $\dfrac{\Delta x}{\Delta t} = D$ – ск-сть дв-я разрыва; $\Delta t \to 0$, Δt

$D - v_1 = u_1$,
$D - v_0 = u_0$

$\rho_1 D - \rho_1 v_1 = \rho_0 D - \rho_0 v_0$; $\rho_1(D - v_1) = \rho_0(D - v_0) \Rightarrow \rho_1 u_1 = \rho_0 u_0$ (1)

$$\int_{t_1}^{t_2}\left(\rho v\big|_{t_2} - \rho v\big|_{t_1}\right)dt + \int_{t_1}^{t_2}\left((\rho v^2 + p)\big|_{x_2} - (\rho v^2 + p)\big|_{x_1}\right)dt = 0$$

$$\left(\rho v\big|_{t_2, x^*} - \rho v\big|_{t_1, x^{**}}\right)\frac{\Delta x}{\Delta t} + (\rho v^2 + p)\big|_{t^*}^{x_2} - (\rho v^2 + p)\big|_{t^{**}}^{x_1} = 0$$

$$(\rho_1 v_1 - \rho_0 v_0)D + \rho_0 v_0^2 + p_0 - (\rho_1 v_1^2 + p_1) = 0$$

$$\rho_1 v_1 D - \rho_1 v_1^2 - p_1 = \rho_0 v_0 D - \rho_0 v_0^2 - p_0; \quad \rho_1 v_1 (D - v_1) - p_1 = \rho_0 v_0 u_0 - p_0 \Rightarrow$$

$$\Rightarrow -\rho_1 u_1 v_1 + p_1 = -\rho_0 u_0 v_0 + p_0; \quad \rho_0 u_0 D = \rho_1 u_1 D; \quad \rho_1 u_1 = \rho_0 u_0;$$

$$\boxed{\rho_1 u_1^2 + p_1 = \rho_0 u_0^2 + p_0} \quad (2)$$

$$\int_{t_1}^{t_2} \left(\rho\left(\varepsilon + \frac{v^2}{2}\right)\Big|_{b_2} - \rho\left(\varepsilon + \frac{v^2}{2}\right)\Big|_{b_1} \right) dt + \int_{b_1}^{b_2} \left(\rho v\left(\varepsilon + \frac{v^2}{2}\right) + pv \right)\Big|_{t_2} - \left(\rho v\left(\varepsilon + \frac{v^2}{2}\right) + pv \right)\Big|_{t_1} d$$

$$\left(\rho_1\left(\varepsilon_1 + \frac{v_1^2}{2}\right) - \rho_0\left(\varepsilon_0 + \frac{v_0^2}{2}\right)\right)D + \rho_0 v_0\left(\varepsilon + \frac{v_0^2}{2}\right) + p_0 v_0 - \rho_1 v_1\left(\varepsilon_1 + \frac{v_1^2}{2}\right) - p_1 v_1 = 0$$

$$\rho_1 \varepsilon_1 D + \rho_1 \frac{v^2}{2} D - \rho_1 v_1 \varepsilon_1 - \rho_1 v_1 \frac{v_1^2}{2} - p_1 v_1 = \rho_1 u_1 \varepsilon + \rho_1 u_1 \frac{v_1^2}{2} - p_1 v_1 =$$

$$= \rho_1 u_1 \varepsilon - p_1 (D - u_1) + \rho_1 u_1 \left(\frac{D^2 - 2Du_1 + u_1^2}{2} \right) = \rho_1 u_1 \varepsilon + p_1 u_1 + \rho_1 u_1 \frac{D^2}{2} - \rho_1 u_1 D u_1 +$$

$$+ \rho_1 u_1 \cdot \frac{u_1^2}{2}$$

$$\rho_1 u_1 \left(\varepsilon_1 + \frac{u_1^2}{2} + \frac{p_1}{\rho_1} \right) = \rho_0 u_0 \left(\varepsilon_0 + \frac{u_0^2}{2} + \frac{p_0}{\rho_0} \right) \quad (3)$$